USING A SPATIAL ENDOGENOUS METHOD TO DETECT HOUSING SUBMARKETS: AN APPLICATION TO TUCSON

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ABSTRACT

While tools modelling spatial autocorrelation have been unanimously adopted in the housing prices literature, there is still no consensus on the appropriate methodology to identify submarkets, i.e. on how to count for spatial heterogeneity. In this paper we propose an innovative methodology that endogenously detects submarkets while counting for spatial autocorrelation across housing prices. The advantage of an endogenous detection is to avoid arbitrariness in the sense that submarkets are defined by the variables of our model only. We apply our methodology to Tucson’s housing market for which our results provide a strong evidence of spatial heterogeneity.

Keywords: Hedonic model, Heterogeneity, Spatial econometrics
1. INTRODUCTION

The importance of location as a determinant of housing prices is widely recognized and housing market researchers are increasingly incorporating spatial effects into hedonic house price models. While the method to account for spatial dependence is generally agreed upon, there is no consensus regarding the best way to account for spatial heterogeneity. Spatial heterogeneity concerns variation in parameter estimates over space. Housing researchers have long recognized that housing parameters may not be constant within metropolitan areas due to inelastic supply and demand of housing characteristics that are spatially concentrated. A failure to incorporate spatial heterogeneity may result in biased parameter estimates and obscure important housing market dynamics. Researchers have long recognized that parameter estimates may vary between housing submarkets (Straszheim, 1974; Schnare and Struyk, 1976). More recent methods that allow housing parameter estimates to vary continuously over space also find evidence for spatial heterogeneity within housing markets (Bitter et al., 2007).

This paper proposes an innovative methodology that endogenously detects submarkets while accounting for spatial autocorrelation across housing prices. The advantage of an endogenous detection is to avoid arbitrariness in the sense that submarkets are defined by the variables of our model only. We apply our methodology to Tucson’s housing market for which our results provide a strong evidence of spatial heterogeneity.

The remainder of the paper is organized as follows: A review of the relevant literature on housing market segmentation is presented in Section 1. Section two describes our model and data while section three develops the spatial endogenous methodology. Section four presents the estimation results and the final section draws conclusions and avenues for further research.

2. DEFINING HOUSING SUBMARKETS: A LITERATURE REVIEW

The importance of location in determining housing prices is universally recognized, but many hedonic price studies have failed to adequately consider the spatial complexity of housing markets (Palmquist, 2005). Two key elements of spatial structure, spatial dependence and spatial heterogeneity, are recognized in the econometrics literature (Anselin, 1988). There are strong reasons to expect that both effects will characterize housing markets. The issue of spatial dependence has received growing attention in the literature and the application of the technique has gained consensus (Pace and Gilley, 1997; Kim et al., 2003; Beron et al., 2004; Brasington and Hite, 2005; Anselin and LeGallo, 2006; Anselin and Lozano-Garcia, 2007). However, the best way to deal with spatial heterogeneity is still an unanswered question.

The issue of spatial heterogeneity centers on whether the marginal prices of housing attributes are constant throughout a metropolitan area or whether they vary with locational context (Orford, 1999). If marginal
implicit prices vary with geographic context, then a failure to segment the model or otherwise incorporate parameter variation may result in biased parameter estimates.

There is good reason to expect the prices of housing site and structural attributes (including environmental features) to exhibit spatial heterogeneity within large housing markets due to localized supply and demand imbalances (Goodman, 1998). Demand by households for specific structural and locational attributes is known to vary based on socioeconomic status, household status, race and ethnicity (Quigley, 1985). Due to the phased nature of the urban development process the supply of specific housing characteristics often exhibit strong spatial patterns within a metropolitan area. Housing is a unique good due to its fixed location and durability, and the characteristics of the housing stock within a particular area may be difficult to change in response to changing demands.

Household budgets and the location of household activities such as the work place may constrain individual households from participating in all segments of a large market. In addition, access to information and the “gate keeping” role played by market participants such as realtors, lenders, and appraisers may also constrain participation (Michaels and Smith, 1990). Thus all housing within a large metropolitan area will not be substitutable and independent hedonic price schedules may arise.

There is no general consensus regarding the best manner in which to incorporate spatial heterogeneity into hedonic house price models. Two broad categories of approaches have been used, which differ conceptually in the manner in which marginal implicit prices vary over space. The first approach allows parameter estimates to vary continuously over space by using variants of the expansion method which involves interacting housing characteristics with neighborhood attributes or absolute location (Theriault et al., 2003, Fik et al., 2003, Bitter et al., 2007) or by estimating a set of unique parameter estimates at each observation location using techniques such as geographically weighted regression (Bitter et al., 2007; Pavlov 2000). These studies find strong evidence that marginal implicit price estimates vary within housing markets.

The second approach, market segmentation, posits that parameter estimates vary between discrete regions within a metropolitan housing market (Straszheim 1974; Schnare and Struyk 1976; Can 1990; Goodman 1998; Michaels and Smith 1990; Bourassa et al., 2003). This form of spatial heterogeneity may arise due to inelastic demand for characteristics such as school quality or municipal services. For instance, inelastic demand for good schools may result in differing marginal implicit prices between adjacent school districts. Advocates of this idea attempt to segment the housing market into discrete submarkets, which are typically defined as geographic areas with similar price structures, and to estimate separate hedonic equations for each.

A number of approaches to market segmentation have been applied within the hedonic literature. One utilizes the expert opinions of local real estate market participants to segment the housing market. For example, Michaels and Smith (1990) classified suburban Boston communities into
market segments defined based on a survey of local realtors. The second, and simplest approach, is to use predefined boundaries such as census tracts, real estate reporting districts, zip codes or school districts to represent housing submarkets and to estimate independent hedonic price schedules for each (Can 1990; Fik et al., 2003). These ad hoc approaches have the benefit of simplicity but can not insure that the resulting submarkets are accurately capturing the spatial structure of the housing market.

More sophisticated approaches to segmentation attempt to statistically delineate areas or individual properties into groupings in which parameter estimates are internally consistent. Goodman and Thibodeau (1998) use a hierarchical modeling approach that nests school districts within municipalities in a study of the Dallas housing market. School quality is assumed to be capitalized into the coefficient for dwelling size. Adjacent school districts are grouped into the same submarket if the coefficients for the dwelling size-test interaction term are not significantly different from zero. The results indicate that housing quality plays an important role in determining submarkets. The author’s identify one drawback of this method - submarket definitions may depend upon the spatial starting point employed.

Bourassa et al. (1999) use principal component analysis and cluster analysis to delineate housing submarkets within Sydney and Melbourne. One set of submarkets is defined by grouping local government authority (LGA) areas to submarkets based on the characteristics of these areas. A second set of submarkets is defined by grouping individual properties to submarkets based on characteristics of the LGA’s and the individual dwellings. The author’s estimate hedonic models for each submarket and compare the predictive accuracy of the statistically defined submarkets to realtor defined housing submarkets. They find that all submarket delineations perform better than an unsegmented model, however, with only one exception, the a priori submarket classifications performed as well as those defined based on the statistical methodology.

Goodman and Thibodeau (2003) compare submarket delineations based on aggregations of zip codes, census tracts, and the hierarchical method developed by Goodman and Thibodeau (1998). They find that all three methods perform better than a “pooled” model with all possible prediction criteria. The more sophisticated statistical method generally out performs the ad hoc specifications in terms of prediction accuracy, but by only a narrow margin.

In sum, there is no generally accepted method to delineate housing submarkets. The statistical methods applied to date have produced submarket delineations that are only slightly superior to ad hoc methods. Moreover, we are not aware of any market segmentation studies that also consider spatial dependence.
3. MODEL AND DATA

The model we use is based on previous work by Bitter et al. (2007) on Tucson’s housing market. The model they use is as follows:

\[
\text{houseprice} = a_0 + a_s \text{sqft} + a_l \text{lotsize} + a_q \text{quality} + a_a \text{age} + a_s \text{story} + a \text{factor1} + a \text{factor2} + \varepsilon
\]

with \( \varepsilon \sim N(0, \sigma^2 I) \) (1)

Where housing prices are expressed in log terms, \textit{sqft} is the square footage, \textit{lotsize} is the size of the lot, \textit{quality} is an index of quality, \textit{age} represents the age, \textit{story} is the number of floors, and \textit{factor1} and \textit{factor2} are the results of a principal component analysis performed in Bitter et al. (2007) on 8 variables (the number of bathroom fixtures per room in the house, the presence of refrigerated air conditioning, the presence of a swimming pool, the total number of rooms divided by dwelling size, the quality and the age of the dwelling, the number of patios, the presence of an enclosed garage). Overall, while \textit{factor1} represents homes with modern features, \textit{factor2} represents another specific style of housing, with a spacious design and outdoor amenities. As usual, \( \varepsilon \) is an error term.

The matrices we use are based on the number of \( k \) nearest neighbors, with \( k=10, 15, 20 \) neighbors. Each matrix is row standardized so that it is relative and not absolute distance which matters. In addition, we use the inverse of the square distance between observations in order to reflect a gravity model. The weight matrices can therefore be written as follows:

\[
\begin{cases}
  w_{ij}(k) = 0 & \text{if } i = j \\
  w_{ij}(k) = 1 / d^2 & \text{if } d_{ij} \leq D_i(k) \\
  w_{ij}(k) = 0 & \text{if } d_{ij} > D_i(k)
\end{cases}
\]

for \( k = 10, 15, 20 \)

Where \( d_{ij} \) is the great circle distance between centroids of region \( i \) and \( j \). \( D_i(k) \) is the critical cut-off distance defined for each region \( i \), above which interactions are assumed to be negligible. \( D_i(k) \) is the \( k^{th} \) order smallest distance between regions \( i \) and \( j \) such that each region \( i \) has exactly \( k \) neighbors. The choice of the weight matrix is somehow always arbitrary. Therefore, in order to test the robustness of our results, we build three other weight matrices based on the great circle distribution and three different distance cut-offs.

We start with the OLS estimation of model (1). Estimation results displayed in column 1 of table 1 show that all the variables have the same sign as in the Bitter et al. (2007) study which also deals with Tucson’s housing market. All the variables but \textit{age} are significant. The \textit{quality} of the house seems to be the most important factor in determining housing prices.
Looking at the diagnostic tests, the Jarque-Bera test rejects the assumption of normality of the residuals (p-value = 0.000). This is due to the presence of spatial effects which will be identified below. We note also that the White test clearly does reject homoskedasticity (p-value = 0.000) as well as the Koenker-Bassett test (p-value = 0.000).

We use Anselin (1988) and Anselin et al. (1996) tests to detect the presence of spatial effects. In order to identify the form of the spatial dependence (spatial error model or spatial lag), the Lagrange Multiplier tests (resp. LMERR and LMLAG) and their robust version are performed. The decision is subject to Anselin and Florax (1995) rule: if LMLAG (resp. LMERR) is more significant than LMERR (resp. LMLAG) and R-LMLAG (resp. R-LMERR) is significant whereas R-LMERR (resp. R-LMLAG) is not, then the most appropriate model is the spatial lag model (resp. the spatial error model).

As seen in table 1, all the Lagrange Multipliers and their robust version are significant. However, the value of the LM (and robust LM) for the spatial error model is greater. This result is confirmed with other weight matrices. Thus, this is the form of spatial autocorrelation we adopt. This is a quite common form in the hedonic literature. Indeed, Pace and Gilley (1997) as well as Beron et al. (2004) use a spatial error model to account for the influence of neighboring houses on housing prices.

The model can be written as follows:

\[
\text{houseprice} = a_0 + a_1\text{sqft} + a_2\text{lotsize} + a_3\text{quality} + a_4\text{age} + a_5\text{story} + a_6\text{factor1} + a_7\text{factor2} + \varepsilon
\]

with \( \varepsilon = \lambda W\varepsilon + e \) and \( e \sim N(0, \sigma^2 I) \) (2)

Where all the variables have the same meaning as before, and the matrix \( W \) is based on the 10 nearest neighbors. Following Bernat’s (1996) interpretation of a spatial error model, this form of spatial autocorrelation indicates that the price of a house is affected by the price of neighboring houses only to the extent that neighboring houses have above or below normal prices.

The second column of table 1 shows the estimation results of model (7) by ML (those results are confirmed by GMM-two steps). In this case, all the coefficients are significant. The age and the number of floors of a house still influence negatively its price. Quality is still the variable which influences prices the most. The coefficient of the spatial error term is 0.774 and is highly significant, indicating that the presence of positive spatial autocorrelation. The two tests against heteroskedasticity (the unadjusted and spatially adjusted Breusch-Pagan statistics) are significant (p-value = 0.000) indicating the presence of remaining heteroskedasticity. The LR-test on the spatial autoregressive coefficient \( \hat{\lambda} \) is highly significant (p-value = 0.000), indicating that the spatial error model is indeed the appropriate specification.
### Table 1: Estimation results of our hedonic model with 𝑾=10

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<tr>
<th></th>
<th>OLS</th>
<th>ML</th>
<th>Diagnostic tests</th>
<th>OLS</th>
<th>ML</th>
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<tr>
<td>Constant</td>
<td>11.000 (0.000)</td>
<td>11.171 (0.000)</td>
<td>J-B test on normality</td>
<td>210.602</td>
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<td>K-B test on heteroskedasticity</td>
<td>142.901</td>
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<td>Sqft</td>
<td>4.29.10^-4 (0.000)</td>
<td>0.37.10^-4 (0.000)</td>
<td>White test</td>
<td>188.579</td>
<td>-</td>
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<tr>
<td>Lot size</td>
<td>3.06.10^-8 (0.000)</td>
<td>3.58.10^-8 (0.000)</td>
<td>Moran’s I</td>
<td>8.366</td>
<td>-</td>
</tr>
<tr>
<td>Quality</td>
<td>0.132 (0.000)</td>
<td>0.091 (0.000)</td>
<td>LM (error)</td>
<td>651.971</td>
<td>-</td>
</tr>
<tr>
<td>Age</td>
<td>-1.3.10^-4 (0.823)</td>
<td>-0.004 (0.000)</td>
<td>Roust LM (error)</td>
<td>407.982</td>
<td>-</td>
</tr>
<tr>
<td>Story</td>
<td>-0.121 (0.000)</td>
<td>-0.092 (0.000)</td>
<td>LM (lag)</td>
<td>341.857</td>
<td>-</td>
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<tr>
<td>Factor 1</td>
<td>0.112 (0.000)</td>
<td>0.051 (0.000)</td>
<td>Robust LM (lag)</td>
<td>97.868</td>
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</tr>
<tr>
<td>Factor 2</td>
<td>0.077 (0.000)</td>
<td>0.056 (0.000)</td>
<td>B-P test on heteroskedasticity</td>
<td>371.434</td>
<td>-</td>
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<tr>
<td>𝜆</td>
<td>-</td>
<td>0.774 (0.000)</td>
<td>Spatial B-P test on heteroskedasticity</td>
<td>371.617</td>
<td>-</td>
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<tr>
<td>LIK</td>
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<td>AIC</td>
<td>-519.556</td>
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<td>LR test on spatial error dependence</td>
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<td>-</td>
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<tr>
<td>SC</td>
<td>-480.302</td>
<td>-949.332</td>
<td></td>
<td>469.029</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes: see notes table 2. IV stands for Instrumental Variables.

### 4. SPATIAL ENDOGENOUS METHOD

The significant results of the B-P tests against heteroskedasticity in table 1 may come either from the presence of structural instability, groupwise heteroskedasticity or both. We will start with a focus on the first form of spatial heterogeneity, more especially on the determination of the clubs at the origin of structural instability. As described in section 1, various methodologies have been used in the literature to consider spatial heterogeneity in hedonic models. The reason for which we do not follow any of them is because they do not pay attention to the spatial effects described in section 2. Indeed, because of the important geographical component of the data upon which our analysis is based, we want the methodology we use for the detection of spatial heterogeneity to take spatial dependence into account.

The methodology we use here combines a spatial approach with an endogenous club detection based on the work of Berthelemy and Varoudakis (1996) who apply it to economic growth and the detection of per capita income clubs. In order to avoid the a priori exogenous choice of the number of clubs as in Durlauf and Johnson (1995), Berthelemy and Varoudakis (1996) perform successive F-tests on coefficients stability (Chow tests) on the entire sample by moving the sample break’s point forward by one observation each time. However, when the first club has been detected, they should repeat their process on the remaining sample to verify whether it is also composed of two sub-groups. Finally, the degree of homogeneity...
between the first and all the other groups remains to be analyzed. This is what we propose in this methodology which is based on previous work by Dall’erba et al. (2007). In addition to the topic of application, the difference with this work relies in the systematic estimation of the degree of homogeneity between successive regimes (i.e., regime 1 with regime 2 and 3, not only with regime 3).

The successive steps of our detection method are as follows:
1) We sort the entire sample according to the increasing distance from the most South-West location.
2) We estimate model (2) with spatial regimes (structural instability treated with dummy variables per regime in model 2) defined as follows: regime 1 is made of the 26 houses near the most South-West location (in order to have a sufficient degree of freedom and because the cross-product of spatially weighted explanatory variables is singular with a lower number of observations in regime 1), regime 2 is made of the other houses.
3) We perform the spatial Chow-Wald test (developed by Anselin, 1995) also called F-test on the overall stability and add one more house in regime 1 (therefore on eless in regime 2) if the test reveals stability between regimes (at 5% significance level).
4) As soon as the Chow-Wald test reveals instability, the regime 1 houses are eliminated from the sample and steps 2 to 4 are repeated on the remaining sample. Observations are re-organized according to their distance to the new most South-Western (or most Southern) observation, and a new weight matrix, still based on k-10 nearest neighbors, is built in order to match the size of the new sample.
5) If multiple regimes are found (say regimes 1, 2 and 3), we need to test how the coefficients of regime 1 are similar to those of regime 2 and 3.

Table 2 below reports the results of the Chow-Wald test of overall stability. We are aware that the process we describe here must be taken with caution because the recursive properties of this test are unknown at finite distance. All the calculations rely on a k-10 nearest neighbors, but have been confirmed with other weight matrices. All of them are based on the great circle distance between centroids.
<table>
<thead>
<tr>
<th>Regime 1: 51 houses</th>
<th>Overall stability</th>
<th>Overall stability</th>
<th>Overall stability</th>
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<tr>
<td>(0.059)</td>
<td>Regime 7: 15 houses</td>
<td>***</td>
<td>Regime 13: 30 houses</td>
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<td></td>
<td>Regime 8: 301 houses</td>
<td>(0.000)</td>
<td>Regime 14: 84 houses</td>
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<td></td>
<td>Regime 8: 300 houses</td>
<td>(0.000)</td>
<td>Regime 15: 31 houses</td>
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<td></td>
<td>Regime 7: 17 houses</td>
<td>(0.000)</td>
<td>Regime 14: 83 houses</td>
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<td></td>
<td>Regime 8: 299 houses</td>
<td>(0.000)</td>
<td>Regime 15: 32 houses</td>
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<td>Regime 13: 30 houses</td>
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<td>Regime 14: 82 houses</td>
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<td></td>
<td>Regime 14: 84 houses</td>
<td>(0.010)</td>
<td>Regime 15: 31 houses</td>
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</table>

**Notes:** ****: Cross-product of spatially weighted explanatory variables is singular, ***: Singular or not positive definite variance matrix. Note: results are obtained by ML estimation. The Chow – Wald test of overall stability is also based on a spatially adjusted asymptotic Wald statistic, distributed as χ² with 2 degrees of freedom (Anselin 1988).

Appendix A displays a map of the 17 regimes found in table 2.

The results of the Chow–Wald tests above indicate the presence of 17 regimes in our sample. However, as indicated in point 5 above, one needs to test how the coefficients of each regime are different from one another. The results of these tests are displayed in table 3 below. 28 of the 120 results below (in bold) indicate that two regimes are not statistically different one from another. The weight matrices that have been used for the calculation are still based on the k-10 nearest neighbors.
Table 3- Spatial Chow-Wald test results for the regimes defined above.

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</tbody>
</table>

Note: p-value into brackets.

With so many non-significant Chow-Wald test, the question of what regimes should be aggregated first arises. Indeed, the way the aggregation is performed influence our results. Therefore, we propose to aggregate first the two regimes that display the least significant Chow-Wald p-value and to test the degree of homogeneity of this new regime with all the previously existing regimes. This process has been repeated 9 times before all the regimes appear not to be significantly similar with each other. As table 4 below indicates, the final number of regimes is 8. The numbers in the top row (or first column) rely on the regimes’ name displayed in table 3. For instance, previous regimes 1 and 10 now belong to the same regime.

Table 4- Spatial Chow-Wald test results for the regimes defined above.

<table>
<thead>
<tr>
<th></th>
<th>3-6</th>
<th>4</th>
<th>5-9</th>
<th>7</th>
<th>2-8-11-13-14-15-17</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-6</td>
<td>0.031</td>
<td>0.000</td>
<td>0.000</td>
<td>0.003</td>
<td>0.000</td>
<td>0.000</td>
<td>3-6</td>
</tr>
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<td>4</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>4</td>
</tr>
<tr>
<td>5-9</td>
<td>0.023</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>5-9</td>
</tr>
<tr>
<td>7</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>7</td>
</tr>
<tr>
<td>2-8-11-13-14-15-17</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.010</td>
<td>12</td>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>

Note: p-value into brackets.
The 8 regimes defined above are represented in figure B below. The most surprising result is to see that clusters do not follow the basic North-South divide which has often been documented in Tucson, because it is based on housing prices only. Our results indicate that regime 2 (houses located in the downtown area of Tucson) is now in the same group as regimes 8-11-13-14-15-17 which are located in the North, i.e. regroup houses which are, on average, much more expensive. However, regime 2 includes the CBD of Tucson (as well as the UofA?) which are two of the biggest employers in town. Without the presence of activity in these locations, other activities would not take place in other parts of the agglomeration, i.e. it may happen that housing prices in regime 2 determine very closely those in the Northern part of the city.

Another interesting result is the group constituted of regimes 1 and 10. They correspond to the most southern and northern parts of the Tucson area and are located along the I-10. These lots have been developed more recently to answer to the increasing demands in housing by newcomers.

It is obvious from the map below that the houses that belong to one particular regime are not always contiguous. This indicates that some houses may have similar dynamics even if they are not geographically clustered. In the methodology used above, space is controlled for in the determination of the regimes, but it is not the only factor at the origin of the regimes.

**5. ESTIMATION WITH BOTH SPATIAL EFFECTS**

Now that we have clearly defined the regimes that are present in our sample, we turn to a cross-section estimation of model (2) to which we add spatial heterogeneity. Indeed, the significance of the BP and spatial BP tests in table 1 clearly indicates the presence of spatial heterogeneity. This may take the form of spatial regime, groupwise heteroskedasticity or both. Let us start with the estimation of the presence of spatial regimes. The model we estimate can be written as follows:

\[
\text{houseprice} = a_{0i} + a_{1i, sqft} + a_{2i, lotsize} + a_{3i, quality} + a_{4i, age} + a_{5i, story} + a_{6i, factor1} + a_{7i, factor2} + \varepsilon + e \lambda W e
\]

with \(\varepsilon = N(0, \sigma^2 I)\) and \(e \sim N(0, \sigma_i^2 I)\)

with subscript \(i=1\) to 8, according to the regime the region belongs to. The results of this estimation are displayed below.
### Table 5 - Estimation results with spatial regimes (ML estimation)

<table>
<thead>
<tr>
<th>Regime</th>
<th>1-10</th>
<th>3-6</th>
<th>4</th>
<th>5-9</th>
<th>7</th>
<th>2-8-11-13-14-15-17</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Constant</strong></td>
<td>11.117 (0.000)</td>
<td>11.198 (0.000)</td>
<td>11.244 (0.000)</td>
<td>11.271 (0.000)</td>
<td>11.746 (0.000)</td>
<td>11.078 (0.000)</td>
<td>10.899 (0.000)</td>
<td>11.135 (0.000)</td>
</tr>
<tr>
<td><strong>Sqft</strong></td>
<td><strong>3.6510^4</strong> (0.000)</td>
<td><strong>3.1710^4</strong> (0.000)</td>
<td><strong>3.8310^4</strong> (0.000)</td>
<td><strong>3.4410^4</strong> (0.000)</td>
<td><strong>2.6210^4</strong> (0.057)</td>
<td><strong>4.0110^4</strong> (0.000)</td>
<td><strong>2.2710^4</strong> (0.235)</td>
<td><strong>3.4110^4</strong> (0.000)</td>
</tr>
<tr>
<td><strong>Lot size</strong></td>
<td><strong>1.0010^3</strong> (0.000)</td>
<td><strong>5.7910^4</strong> (0.000)</td>
<td><strong>4.5510^4</strong> (0.000)</td>
<td><strong>3.0810^4</strong> (0.000)</td>
<td><strong>2.1410^4</strong> (0.000)</td>
<td><strong>7.4010^4</strong> (0.000)</td>
<td><strong>7.5210^4</strong> (0.164)</td>
<td><strong>2.3010^4</strong> (0.000)</td>
</tr>
<tr>
<td><strong>Quality</strong></td>
<td>-0.024 (0.609)</td>
<td>0.132 (0.000)</td>
<td>0.078 (0.085)</td>
<td>0.069 (0.088)</td>
<td>0.115 (0.165)</td>
<td><strong>0.100</strong> (0.000)</td>
<td>-10.171 (0.171)</td>
<td>-0.030 (0.804)</td>
</tr>
<tr>
<td><strong>Age</strong></td>
<td>-0.007 (0.005)</td>
<td>-0.003 (0.015)</td>
<td>-0.007 (0.001)</td>
<td>-0.001 (0.168)</td>
<td>-0.012 (0.018)</td>
<td>-0.004 (0.000)</td>
<td>-0.003 (0.037)</td>
<td>0.017 (0.783)</td>
</tr>
<tr>
<td><strong>Story</strong></td>
<td>-0.044 (0.338)</td>
<td>-0.030 (0.606)</td>
<td>0.013 (0.798)</td>
<td>-0.063 (0.141)</td>
<td>-0.060 (0.000)</td>
<td><strong>-0.120</strong> (0.000)</td>
<td>-0.045 (0.660)</td>
<td>-0.165 (0.105)</td>
</tr>
<tr>
<td><strong>Factor 1</strong></td>
<td>-0.011 (0.745)</td>
<td>0.036 (0.102)</td>
<td>0.004 (0.895)</td>
<td>0.114 (0.000)</td>
<td>0.045 (0.720)</td>
<td>0.076 (0.000)</td>
<td>0.096 (0.659)</td>
<td>0.040 (0.659)</td>
</tr>
<tr>
<td><strong>Factor 2</strong></td>
<td>0.049 (0.014)</td>
<td>0.031 (0.027)</td>
<td>0.037 (0.082)</td>
<td>0.056 (0.000)</td>
<td>0.012 (0.879)</td>
<td><strong>0.058</strong> (0.000)</td>
<td>0.014 (0.838)</td>
<td><strong>0.108</strong> (0.001)</td>
</tr>
</tbody>
</table>

| λ                        | 0.760 (0.000)             | 0.871 (0.000)            | 585.290 (0.000)           | -1042.58 (0.000)          | -728.548 (0.000)          | 355.207 (0.000)     | 33.954 (0.000)           |

Comparing the results that include spatial heterogeneity with those of table 1, it appears that the fit of the model has improved. The spatial lag coefficient is again positive and very significant. All the coefficients are in the range of what the corresponding coefficient was (see table 1). Square footage is the variable which is the most often significant, while story is significant only once. No coefficient is significant in regime 12 while all the coefficients are significant in regime 2-8-11-13-14-15-17.

Finally, the last model tested includes both spatial regimes and groupwise heteroskedasticity. However, the results of this model are not presented here because the fit of the model is much lower than the one of the previous model and only 10 coefficients are significant.

### 6. CONCLUSION

This paper has brought an innovative way to look at the detection and definition of heterogeneity in the housing market. Indeed, we use an endogenous methodology that pays attention to spatial autocorrelation when identifying sub-markets. This is the first time this methodology is applied on housing prices.

Our results, based on a cross-sectional estimation, shed some light on the dynamics of Tucson’s real estate market. Some groups of houses which are not necessarily located close to each other display similar patterns indicating they belong to the same sub-market. Overall, while Tucson’s housing prices increase from South to North, our results indicate that some
groups of houses on the Easter and Western part of the city show similar
dynamics and that the most historic houses, located in the downtown area
and close to the University of Arizona, display a similar pattern than some of
the expensive and more recent houses located close to the Catalina foothills.
This is an outcome we would have never reached would we have focused
only on housing prices.

REFERENCES


